Statistics

Lecture 14

Feb 19-8:47 AM

Estimating Parameters:

§ 21 & § 22

Parameters describe Population

Statistics " Sample

we use Statistic to guess parameter.

we use $\bar{x}$ to estimate μ.

we use S to estimate σ.

we use $\hat{p}$ to estimate p.

$\hat{p}$ - p-hat, Sample Proportion

p - Population Proportion

$\bar{x}$ is the point-estimate for μ.

S " " " " " σ.

$\hat{p}$ " " " " " p.

Point-estimate is our first/best guess.

Jul 23-4:39 PM

When estimating any parameters, the **answer** will be a **range of values**

Confidence Interval

$84 < \mu < 96$ Conf. interval for Pop. Mean

$8.5 < \sigma < 10.8$ " " " " Standard Dev.

$.38 < p < .52$ " " " Pop. Proportion

Prob. that the parameter falls in the Conf. interval is called Confidence level.

Middle Area in the graph of Prob. dist.

If C-level not given $\Rightarrow$ use 95%

Jul 23-4:46 PM

Study shows the mean age of all nurses is between 32 and 46 with 90% Conf. level.

$P(32 < \mu < 46) = .9$

Study shows the prop. of all voters in support of .... is between 68% to 80% with 98% Conf. level.

$P(.68 < p < .80) = .98$

Prob. Proportion

LA Times says Standard dev. of Salaries of all teachers in LAUSD is between \$380 & \$450.

NO C-level $\Rightarrow$ use 95%

$P(380 < \sigma < 450) = .95$

Jul 23-4:52 PM

Estimating Population Proportion

$$\hat{p} - E < p < \hat{p} + E$$

- $\hat{p}$: Sample Proportion
- p: Pop. Proportion
- E: Margin of error

$$\hat{p} = \frac{x}{n}$$

- x: # of favorable responses
- n: Sample Size

$$\hat{q} = 1 - \hat{p}$$

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$z_{\alpha/2}$: Critical Value for $(1-\alpha)\cdot 100\%$ C-level

Jul 23-5:01 PM

I surveyed 100 students, and 80 of them had iPhone.

$n = 100$ $\quad x = 80$

$\hat{p} = \frac{x}{n} = \frac{80}{100} = .8$ $\quad \hat{q} = 1 - \hat{p} = 1 - .8 = .2$

Find 90% Conf. interval for the Prop. of all students that have iPhone.

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 1.645 \cdot \sqrt{\frac{(.8)(.2)}{100}}$

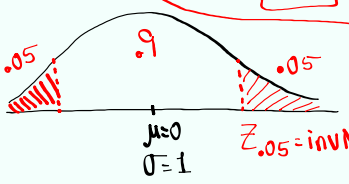
$\approx .07$

$\hat{p} - E < p < \hat{p} + E$

$.8 - .07 < p < .8 + .07$

$.73 < p < .87$

we are 90% confident that between 73% & 87% of all students have iPhone.



$z_{.05} = \text{invNorm}(.95, 0, 1) = 1.645$

C-level : .9

Jul 23-5:07 PM

Using TI:

STAT → TESTS 1 | 1-Prop ZInt

$x = 80$

$n = 100$

C-level: .9

Calculate

$.734 < p < .866$

$.73 < p < .87$

$\hat{p} = \frac{.87 + .73}{2} = \frac{1.6}{2} = .8$

$E = \frac{.87 - .73}{2} = \frac{.14}{2} = .07$

Jul 23-5:18 PM

I surveyed 80 students, and 15 were smokers. $n = 80$ $x = 15$

Find 99% Conf. interval for the prop. of all students that are smokers. → C-level .99

1-Prop ZInt

$x = 15$

$n = 80$

C-level: .99

Calculate

$.08 < p < .30$

we are 99% confident that between 8% & 30% of all students are smokers.

$\hat{p} = \frac{.30 + .08}{2} = .19$

$E = \frac{.30 - .08}{2} = .11$

Jul 23-5:24 PM

I surveyed 185 voters and 42% of them were in support of ICE action.

$n = 185$ $\quad x = n\hat{p} = 185(.42) = 77.7 = 78$

$\hat{p} = .42$ $\quad$ if decimal → Round-up

find Conf. interval for the prop. of all Voters in support of ICE action.

↳ No C-level ⇒ use .95

$.35 < p < .49$

STAT | TESTS | 1-PropZInt

$x = 78$

$n = 185$

C-level: .95

Calculate

with 95% conf. level, Prop. of all Voters in Support of ICE is between 35% & 49%.

$\hat{p} = \frac{.49 + .35}{2} = .42$

$E = \frac{.49 - .35}{2} = .07$

Jul 23-5:31 PM

Estimating Population Mean:

$$\bar{x} - E < \mu < \bar{x} + E$$

$\bar{x}$: Sample Mean

μ: Population Mean

E: Margin of error

Case I: σ known	Case II: σ unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $\quad df = n-1$
STAT TESTS ZInterval	STAT TESTS TInterval
inpt: Stats	inpt: STATS

$E = \frac{-}{2}$ $\qquad \bar{x} = \frac{+}{2}$

Jul 23-5:55 PM

Given $n=32$ $\bar{x}=85$ $\sigma=12$

Find 98% Conf. interval for Pop. Mean

$80 < \mu < 90$

σ known → Z Interval

σ unknown → T Interval

$\bar{x} = \frac{90+80}{2} = 85$

$E = \frac{90-80}{2} = 5$

STAT TESTS

Z Interval

Stats

inpt:

σ : 12

$\bar{x}$: 85

n : 32

C-level : .98

Calculate

Jul 23-6:03 PM

Given $n=15$ $\bar{x}=32.5$ $s=7.5$

Find 90% Conf. interval for the Pop. mean.

$29.1 < \mu < 35.9$

σ Known → Z Interval

σ unknown → T Interval

STAT TESTS T Interval

Stats

inpt:

$\bar{x} = \frac{35.9+29.1}{2} = 32.5$

$E = \frac{35.9-29.1}{2} = 3.4$

$\bar{x}$: 32.5

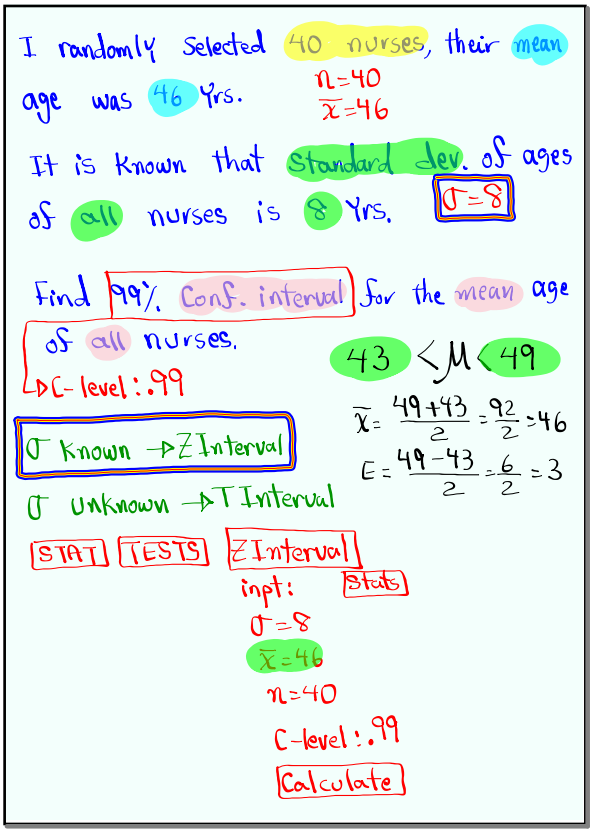
Sx : 7.5

$df = 15-1 = 14$ ← n : 15

C-level : .9

Calculate

Jul 23-6:08 PM



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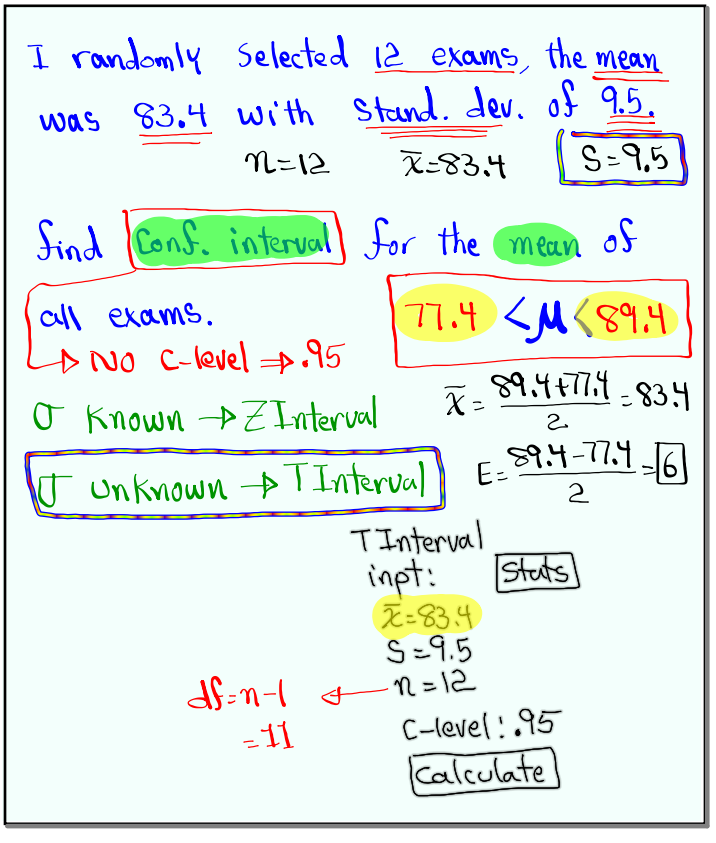


Jul 23-6:25 PM

I randomly selected 10 new textbooks.

Here are the prices:

95	120	100	80	70
150	135	140	100	90

Find $\bar{x}$ & s
Round to whole #.

$\bar{x} = 108$
$s \approx 27$
$n = 10$

C-level: .9
Find 90% Conf. interval for the mean Price of all new textbooks

$92 < \mu < 124$

$\bar{x} = \frac{124 + 92}{2} = 108$

$E = \frac{124 - 92}{2} = 16$

σ unknown

T Interval
inpt: Stats
$\bar{x} = 108$
$s = 27$
$n = 10$
C-level: .9
Calculate

Jul 23-6:35 PM

How to determine minimum Sample Size needed!

1) Population Prop.

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

with some algebra

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

if decimal $\Rightarrow$ Round-up

If $\hat{p}$ & $\hat{q}$ are unknown, use .5 for each

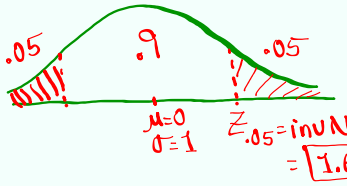
$$n = (.5)(.5)\left(\frac{z_{\alpha/2}}{E}\right)^2$$

$$= .25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

Jul 23-6:46 PM

Find min. Sample Size needed to Construct 90% Conf. interval for Pop. Prop. with margin of error of 5%.

1) Assume $\hat{P}=.4$. $\quad n=\hat{P}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$



$z_{.05}=\text{invNorm}(.95,0,1)$

$=1.645$

$=(.4)(.6)\left(\frac{1.645}{.05}\right)^2$

$=259.7784$

$n=260$

2) Assume $\hat{P}$ Unknown

$n=.25\left(\frac{z_{\alpha/2}}{E}\right)^2=.25\left(\frac{1.645}{.05}\right)^2$

$=270.6025$ $\quad n=271$

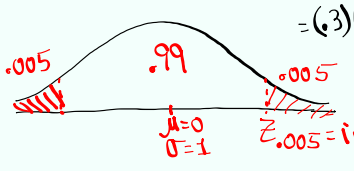
Jul 23-6:50 PM

C-level: .99 $\quad E=4\%$ $\quad$ Find n

1) $\hat{P}=.3$ $\quad n=\hat{P}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$

$=(.3)(.7)\left(\frac{2.576}{.04}\right)^2$

$n=871$



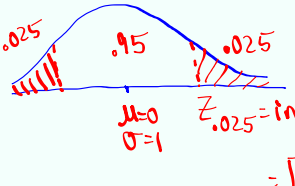
$z_{.005}=\text{invNorm}(.995,0,1)$

$=2.576$

Redo with 95% C-level & $E=8\%$.

$n=(.3)(.7)\left(\frac{1.960}{.08}\right)^2$

$n=127$



$z_{.025}=\text{invNorm}(.975,0,1)$

$=1.960$

For $\hat{P}$ unknown $\quad n=.25\left(\frac{1.960}{.08}\right)^2\approx 151$

Jul 23-6:58 PM

How to determine minimum Sample Size needed:

2) Population Mean

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

with some algebra $\quad n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$

if decimal $\Rightarrow$ Round up

If σ unknown $\Rightarrow$ use s

$$n = \left(\frac{z_{\alpha/2} \cdot s}{E}\right)^2$$

Jul 23-6:46 PM

Find min. Sample Size needed to construct 98% C-level for pop. mean with error not to exceed 10. Assume $\sigma = 8$.

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

$$\left(\frac{2.326 \cdot 8}{10}\right)^2$$

$n = 4$

Redo with $E = 5$

$$n = \left(\frac{2.326 \cdot 8}{5}\right)^2 \approx 14$$

Redo with 99% C-level & $E = 5$

$$n = \left(\frac{2.576 \cdot 8}{5}\right)^2 \approx 17$$

Jul 23-7:10 PM